# A FINITE ITERATION TECHNIQUE FOR A FUZZY QUADRATIC PROGRAMMING PROBLEM

Ashfar Ahmed[#1], Mantha Srikanth[#2], Venkataramana Musala[#3]

[#]*Assistant Professor, Department of Mathematics, Malla Reddy Engineering College (Autonomous), Hyderabad-500100, Telangana, India*

[1]ahmedashfaq02@gmail.com

[2]manthasrikanth9@gmail.com

[3]vramanareddy.m85@gmail.com

***Abstract***— **In this paper, we develop a finite iteration technique to solve a fuzzy quadratic programming problem with single quadratic objective function and a number of linear constraints. The quadratic programming problem has a lot of applications in the field of economics, finance, statistics, and structural engineering. All the methods available to solve fuzzy quadratic programming problems are very lengthy and require high level of knowledge in the field of mathematics. Therefore, method proposed in this paper, is an attempt to provide an easy tool to address this kind of problems.**

***Keywords***—**Uncertainty, Decision making, Finite iteration technique, Symmetric fuzzy quadratic programming, Linear constraints**

## 1.1 INTRODUCTION

Uncertainty is one of the most significant problems that have to be addressed by current management systems. Decision making under uncertainty has become a key issue in the present alternative way of thinking. There is an evolving interest in the usage of novel techniques to draw definite conclusions from imprecise data. A grave challenge in decision-making is to find an appropriate method to measure and quantify the uncertainty involved in the problem under consideration and its effective applications. Fuzzy solutions are used to find out the possible feasible region of decision variables. Fuzzy logic is basically a multi-valued logic that allows intermediate values to be defined between conventional evaluations like yes/no, true/false, etc. As fuzzy reasoning and logic have the capability to express the amount of ambiguity in human thinking and subjectivity in a comparatively undistorted manner, they find their major applications in areas of quantitative analysis, information retrieval control, pattern recognition and inference.

Since Zadeh (1) introduced the concept of fuzzy set theory, a number of researchers have exhibited their interest in the topic of fuzzy mathematical programming (2-5). However, in contrast with the vast literature available on modeling and solution procedures for a linear program in a fuzzy environment, the studies in quadratic programming under fuzzy environment and its solution are scarce. In this article, we consider a graduation problem with imprecise observed values data and imprecise combination of fit and smoothness. The problem is first formulated, solved and analyzed as a fuzzy linear program. Next, a finite iteration technique is developed to solve a fuzzy quadratic programming problem. We present two problems, one under symmetric fuzzy environment, and the second under non-symmetric fuzzy environment, such that each problem has a single quadratic objective function and a number of linear constraints. Each of the two fuzzy problems is converted into a crisp programming problem that has a linear objective function with linear constraints, and has one quadratic constraint. To solve such a problem, we suggest a finite step method that uses linear programming and parametric quadratic programming. Furthermore, we present a numerical example to demonstrate the method developed. Significance of this model can be hopefully seen in the light of usage of quadratic program in the field of Finance, Economics, Structural Engineering and Actuarial Sciences under uncertainty.

## 1.2 A CLASSICAL QUADRATIC PROGRAMMING PROBLEM (6-11)

(P-1) Minimize (1.2.1)
Subject to

$Ax \leq$ b (1.2.2)

$x \geq 0$ (1.2.3)

where each of p and C is a symmetric matrix, A is an matrix and we also assume that the feasible solution set of the constraints is bounded. Further, we assume that the quadratic objective function is pseudo convex.

## 1.3 SYMMETRIC FUZZY QUADRATIC PROGRAMMING

Corresponding to (P-1), we now consider following symmetric fuzzy version (P-2) on the lines of Zimmermann (5).
(P-2) Find a solution that satisfies:

(1.3.1)
i=1, 2, ….., k (1.3.2)
i=k+1, …., m (1.3.3)
(1.3.4)

where the fuzzy inequality ‘’ denotes ‘essentially less than or equal to’, and $z_0$, called the aspiration level, is given some pre-assigned value.
Let, $q_0 > 0$, and $q_i > 0$, (i = 1, 2,…, k), be subjectively chosen of admissible violations such that $q_0$ is associated with (1.3.1), and $q_i$ (i = 1, 2,…, k) are associated with the i-th linear constraint (1.3.2). Now, on the lines of Zimmerman (5), we define the membership function corresponding to (1.3.1) and (1.3.2), as follows.

and

## 1.4 THE EQUIVALENT CRISP PROBLEM

On the lines of Zimmermann (5), the solution to the problem (P-1) is obtained by solving the following problem (P-3).

Maximize Minimize
(P-3) $i \geq$ 0, 1, 2, …., k
Subject to (1.3.3), and (1.3.4).

Now following Schmitendorf (12), (P-3), and Zimmermann (5), a solution to (P-3) is obtained by solving the following problem (P-4).
Maximize
Subject to

i=0, 1, 2,……, k

and (1.3.3), and (1.3.4).

From above, using the expressions for for i=0,1,2,……,k. and using (1.3.3), and (1.3.4), we obtain (P-4) as follows.
(p-4) Maximize
Subject to

, i=1, 2,…… ,k.

i=k+1,……, m.



Rewriting (p-4), we obtain (p-5) as follows.
(p-5) Maximize

Subject to

\+ (1.4.1)

+, i=1,2,……,k. (1.4.2)

(1.4.3)

, i=k+1,……, m (1.4.4)

(1.4.5)

In (p-5), objective and the constraints (1.4.2)-(1.4.5) are linear. However, the constraint (1.4.1) is quadratic. Therefore, (p-5) is of the type of the problem (VP).

## 1.5 NON-SYMMETRIC FUZZY QUADRATIC PROGRAMMING

We now consider the following non-symmetric fuzzy quadratic programming problem (NFP).

(NFP)

Maximize

Subject to

i=1, 2, ….., k

i=k+1,…., m

As suggested by Zimmermann (5), we compute the membership function corresponding to the quadratic objective function with the help of the following two crisp quadratic programs(CP-1) and (CP-2).

(CP-1) Maximize

Subject to

i=1, 2, ….., k

i=k+1,…., m

(CP-2) Maximize

Subject to

i=1, 2,….., k

i=k+1,…., m

Let the minimum value of f(x) be $f_1$ .

Then, on the lines of Zimmermann (5), the membership function corresponding to the quadratic objective of (NFP) is defined as follows.

Now, the equivalent crisp programming problem corresponding to (NFP) is as follows.

Maximize $x_{n+1}$

Subject to

$P^{T}x + x^{T}C x + (f_1-f_0)\, x_{n+1} \leq f_1$

$+ q_i x_{n+1} \leq q_i + d_i$, i =1,2,…….k.

$x_{n+1} \leq 1$

$\leq b_i$, i=k+1,………..m.

$x , x_{n+1} \geq 0$

which is similar to (P-5), and therefore, can be solved on the lines of the Two-Phase method suggested for solving (P-5).

## 1.6 NUMERICAL EXAMPLE

We now solve a numerical example for the following fuzzy symmetric quadratic programming problem (FSQP) using the method described above.

(FSQP) $2\ x_1 + 1\ x_2 + 4\ x_1^2 + 4\ x_1\ x_2 + 2\ x_2^2 51.88$

$4x_1 + 5x_2\ 20$

$5\ x_1 + 4\ x_2 20$

$1\ x_1 + 1\ x_2\ 30$

$x_1, x_2\ 0.$

Let $q_0 = 2.12$, $q_1 = 2$, $q_2 = 1$, $q_3 = 3$.

Then, on the lines of (P-5), the crisp equivalent of this problem is

Maximize $x_3$

Subject to

$2\ x_1 + 1\ x_2 + 4\ x_1^2 + 4\ x_1 x_2 + 2\ x_2^2 + 2.12\ x_3 \leq 54$

$4\ x_1 + 5\ x_2 - 2\ x_3 \geq 18$

$5\ x_1 + 4\ x_2 - 1\ x_3 \geq 19$

$1\ x_1 + 1\ x_2 + 3\ x_3 \leq 33$

$x_3 \leq 1$

$x_1, x_2, x_3 \geq 0.$

This problem is similar to (VP) with linear objective function, exactly one quadratic constraint and three linear constraint. Therefore, we solve it in a finite number of steps using the Two-Phase method as outlined above for solving (VP).

In Phase 1, the linear programming problem is as follows.

Maximize $x_3$

Subject to

$4\ x_1 + 5\ x_2 - 2\ x_3 \geq 18$

$5\ x_1 + 4\ x_2 - 1\ x_3 \geq 19$

$1\ x_1 + 1\ x_2 + 3\ x_3 \leq 33$

$x_3 \leq 1$

$x_1, x_2, x_3 \geq 0.$

Its optimal solution is $x_1 = 2.2222$, $x_2 = 2.2222$, $x_3 = 1$.

Since $2\ x_1 + 1\ x_2 + 4\ x_1^2 + 4\ x_1\ x_2 + 2\ x_2^2 + 2.12\ x_3 = 58.17$ at

$x_1 = 2.2222$, $x_2 = 2.2222$, and $x_3 = 1$, therefore the constraint

$2\ x_1 + 1\ x_2 + 4\ x_1^2 + 4\ x_1 x_2 + 2\ x_2^2 + 2.12\ x_3 \leq 54$ is violated. Hence we go Phase 2. In this phase we solve the following quadratic programming problem parametrically.

Minimize $2\ x_1 + 1\ x_2 + 4\ x_1^2 + 4\ x_1 x_2 + 2\ x_2^2 + 2.12\ x_3$

Subject to

$4\ x_1 + 5\ x_2 - 2\ x_3 \geq 18$

$5\ x_1 + 4\ x_2 - 1\ x_3 \geq 19$

$1\ x_1 + 1\ x_2 + 3\ x_3 \leq 33$

$x_3 \leq 1$

$x_3 \geq 1$

$x_1, x_2, x_3 \geq 0.$

From this problem, by solving a series of quadratic programs parametrically, we obtain the final form of the quadratic programming as follows.
Minimize $2 x_1 + 1 x_2 + 4 x_1^2 + 4 x_1 x_2 + 2 x_2^2 + 2.12 x_3$

Subject to

$4 x_1 + 5 x_2 - 2 x_3 \geq 18$

$5 x_1 + 4 x_2 - 1 x_3 \geq 19$

$1 x_1 + 1 x_2 + 3 x_3 \leq 33$

$x_3 \leq 1$

$x_3 \geq 0.860633$

$x_1, x_2, x_3 \geq 0.$

The optimal solution to this problem is $x_1 = 0.99$, $x_2 = 3.73$, $x_3 = 0.860633$, and the minimum value of the objective function is = 54.

Thus, the solution that solves the (FSQP) is

$x_1 = 0.99$, $x_2 = 3.73$,

and the level of satisfaction of this solution is given by $x_3 = 0.860633$.

## 1.7 CONCLUSION

In the present paper, we consider a symmetric fuzzy quadratic programming problem. Solution to this problem is obtained in a finite number of steps by solving an optimization problem in which one constraint is quadratic, other constraints and the objective function are linear. Also, it is shown that the non-symmetric fuzzy quadratic programming problem can also be solved in a finite number of steps by using a similar technique.

## REFERENCES

1. Zedah, LotfiA., “Information and control”, Vol.8, 1965, pp.338-353.
2. Bector,C.R., and Chandra,S., “On duality in Linear Programming Under Fuzzy Environment”, Management Science, 17, 1970, pp. 141-164.
3. Hamacher, H., Liberling, H., and Zimmermann,H.-J., :Sensitivity Analysis in Fuzzy Linear Programming’, Fuzzy Sets and Systems,1 (1978), pp.269-281.
4. Lai, Joung-Jou, and Hwang, Ching Lai, “Fuzzy MathematicakProgeramming”, Springer Verlag, Berlin, 1992.
5. Zimmermann, H.-J, ”Fuzzy set theory and its Applications” Kluwer-Nijhoff Publishing, Second edition 1991.
6. Beale, E.M.L. “On Quadratic Programming”, Naval Research Logistic Quaterly 6, 1959, pp.227-243.
7. Martos, B., “Quadratic Programming with a Quasiconvex Objective Function”, Operation Research 19, 1971, pp. 87-97.
8. Mylander, W. C., “Finite Algorithms for Solving Quasiconvex Quadratic Programs”, Operations Research 20, 1972, pp.167-173.
9. Van de Panne, C., and Whinston, A., “The Simplex and the Dual Method for Quadratic Programming”, “, Operational Research Quarterly 15, 1964, pp.355-388.
10. Van de Panne, C., “Programming with a Quadratic Constraint”, Management Science 12, 1966, pp.798-815.
11. Wolfe, Philip, “The simples Method for quadratic Programming”, Econometrica, Vol.27, No. 3, July, 1959, pp.382-398.
12. Schmitendorf, W.E., “A Simple Derivation of necessary Conditions for Static Minimax Problems”, Journal of Mathematical Analysis and applications, 1979 (70), pp. 486-489.